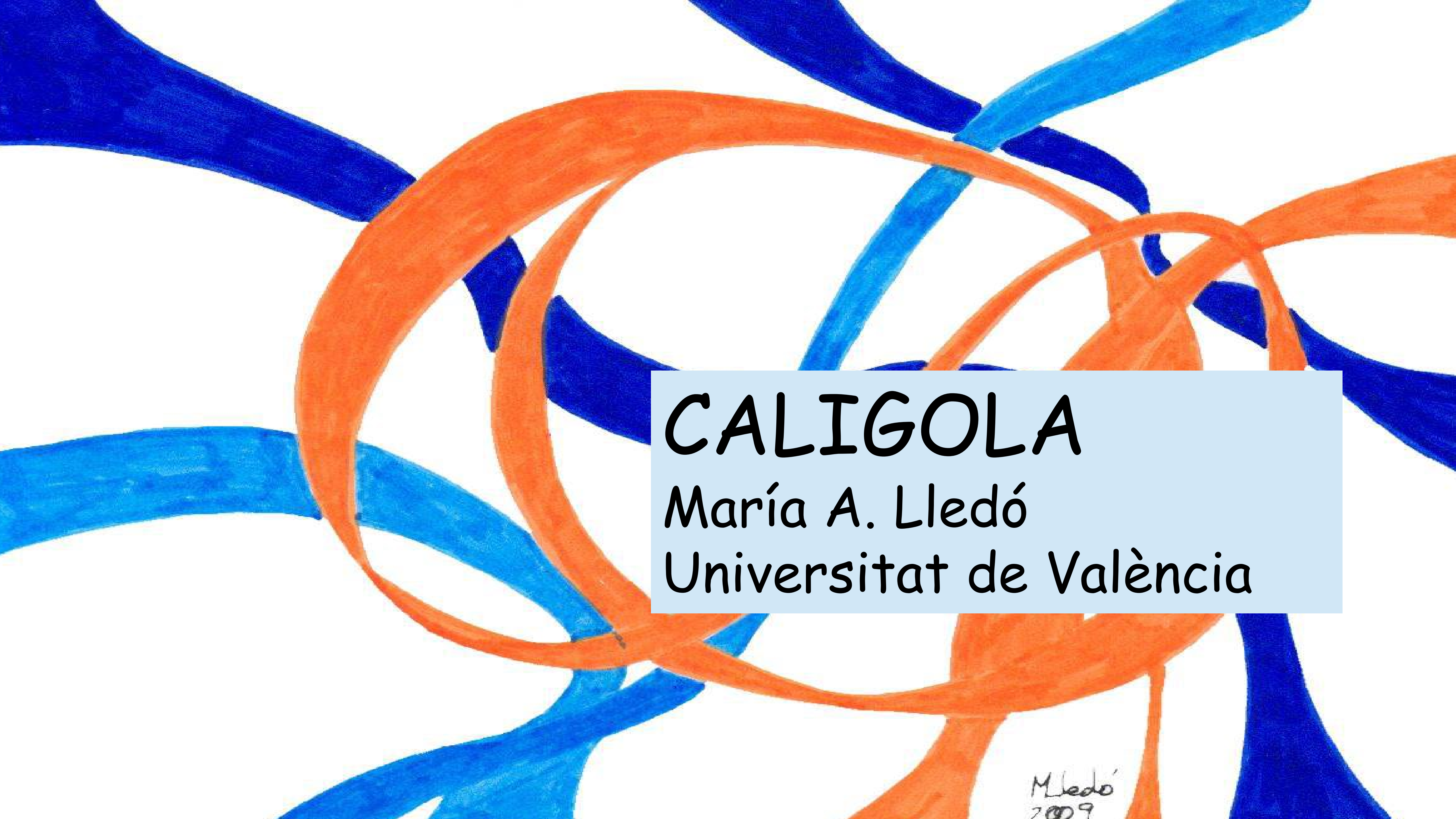
CALIGOLA

María A. Lledó
Universitat de València



RESEARCH

- Quantization of conformal and Minkowski superspaces
- Continuous spin in supersymmetry
- Torsion gravity and supergravity in the Cartan formalism
- Quantum logic and fuzzy logic

Quantization of conformal and Minkowski superspaces in D=4

The conformal group in D=4 is SO(2,4). Its spin group is SU(2,2) and its complexification in SL(4,C). As a subgroup of SL(4, C) we have the Poincaré group. The complex Grassmannian G(2,4) is an homogeneous space for the group SL(4, C), since it acts on C^4 and sends a two plane in another two plane transitively. C^4 is the twistor space. The big cell of G(2,4) is isomorphic to the four dimensional, complexified Minkowski space (also C^4, since the Poincare subgroup acts on it in the expected way.

The picture carries over to the four dimensional, complexifed, N-superspace. The superconformal group is SU(2,2|N) and as a subgroup we have the super Poincaré group. Complexifying, we have SL(4|N) and several homogeneous spaces that are the different superspaces. We have the chiral and antichiral superspaces G(2|N, 4|N) and G(2|0, 4|N). For N=1 we have the real superspace (complexified) Fl(2|0, 2|1, 4|1). For N>1, this space can be constructed but it is too big to be of use in physics. These are all projective varieties and are given in terms for generators and relations by the super Plücker relations. In the case of the flag we have to add twistor relations. It happens that all these supervarieties are projective.

We can quantize this construction by substituting SL(4|N) by SLq (4|N) and defining the quantum conformal and Minkowski spaces properly. The projectivity is crucial in this generalization. We have quantized the chiral and antichiral superspaces for N=1,2.

In physics it appears another superspace with physical relevance, the harmonic superspace. It allows to impose covariant constraints by extending the reduced part of the supermanifold. The simplest one is $G(2|,4|2)$. In principle, on could quantize, but we don't have the help of the projectivity to guide us in defining the quantum coordinates.

We have classified all the harmonic superspaces with underlying D=4 Minkowski spacetime as bigcell and computed the twistor relations.

All these considerations have lead us to the theory of invariants of $SL(m|n)$ and to generalize the first and second theorems of invariant theory.

Continuous spin in supersymmetry

Representations of zero mass of the Poincaré group with little group E(2) (the last Wigner particle). E(2) is also non compact and a direct product, so its unitary representations are infinite dimensional. This particle has not been found in Nature (but when does this have stopped us?).

These representations exist also for super Poincaré. We plan to investigate them using the theory of induced representations, as given by Mackey.

There appear to be a tower of higher spin fields (or multiplets), but with a non triviall action of the Poincare group, that mixes them.

What is the relation with standard higher spin theories? Are the standard ones a limit, somehow, of the continuous spin theories? What would be a physical theory for the last Wigner particle?

Torsion gravity and supergravity in the Cartan formalism

Cartan connections have, generically, torsion, unlike Riemannian connections. Notably, in theories like tereparallel gravity torsion is used to carry the degrees of freedom of the curvature by considering a torsionfull connection which is flat. The spacetime is required to be parallelizable.

One can go a step further: first maintaining the Cartan formalism with p-forms in the formulation of the gravity (adequate to couple it to spinors). Then by switching on the torsion without necessarily turning off the curvature. This doesn't increase the number of degrees of freedom if the part of the connection which has curvature is seen as a kind of background. It is a generalization of teleparallel gravity, using a more general connection and not assuming the parallelizability of spacetime.

Quantum logic and fuzzy logic

Quantum logic is a way of making quantum mechanics more abstract and, consequently, simpler. As the postulates of Mackey in physics, the idea is to depart from some simple assumptions to obtain 'something' related with quantum mechanics. There is a full family of logics that can be called 'quantum' (and, in fact, classical logic is one of them). A quantum logic does not necessarily satisfy the distributive axiom. It has been proven that a quantum logic is equivalent to a fuzzy logic, in which sets are such that elements have a 'degree of membership' between 0 and 1.

Quantum logics satisfies all the properties stated in Mackey's axioms except one: the logic has to be the Birkhoff von Neumann logic of Hilbert spaces.

This is often regarded as an ad-hoc postulate.

What does this requirement do to fuzzy logic?


THANK YOU
M Lledó
2009